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Mobile impurity in one-dimensional correlated electron systems*

Y. Tsukamoto, T. Fujii, and N. Kawakami[a]

Department of Applied Physics, Osaka University, Suita, Osaka 565, Japan

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Abstract. Critical properties of one-dimensional (1D) correlated electron systems with a mobile impurity are investigated. By applying the finite-size scaling method to a Bethe-ansatz solvable model, we derive the conformal dimensions related to the orthogonality catastrophe. We then apply the results to the Fermi-edge singularity in quantum wires, and clarify how the critical exponent for X-ray absorption depends on the mass of the core-hole created. A generalization to SU(ν) electron systems is outlined based on the g-on description of 1D electron systems.

PACS. 71.10.Pm Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, etc.) – 72.15.Nj Collective modes (e.g., in one-dimensional conductors) – 72.10.Fk Scattering by point defects, dislocations, surfaces, and other imperfections (including Kondo effect)

1 Introduction

Critical phenomena in one-dimensional (1D) correlated electron systems have been providing one of the most attractive subjects in condensed matter physics. Low-energy properties for such correlated electron systems are generally described by Tomonaga-Luttinger (TL) liquids [1]. More recently, TL liquids with some impurities or defects have attracted considerable attention. For example, in connection with the Coulomb blockade in quantum wires the effect of a localized impurity has been studied in detail by means of perturbative renormalization group method [2–10]. In this connection, various impurity models for 1D correlated electron systems have been studied extensively [11–20].

So far, the impurity effect on TL liquids has been mainly studied for a *static impurity*. In this paper, we wish to address the question how a *mobile impurity* affects the critical properties of 1D correlated electron systems. This problem is interesting because it is related not only to the problem of a heavy particle in metal [21–23] but also to the X-ray absorption experiments in quantum wires [24], where a core hole created is regarded as a mobile impurity. In particular, motivated by the above experiments, extensive studies on the spectral properties have been done theoretically. By using bosonization approach, Ogawa *et al.* concluded that the Fermi-edge singularity (FES) exponent may not depend on the mass of a core hole [5]. On the other hand, Castella and Zotos claimed that the FES exponent should take different values at least between two non-trivial limits of core-hole mass [25,26]. In the previous paper, as a first step to this type of impurity problem, we have investigated the critical properties of 1D boson system with a mobile impurity [27]. In this paper, by extending our analysis to correlated electron systems, we derive some consequences about the effect of a mobile impurity on electron systems. We also generalize the analysis to SU(ν) electron systems.

The organization of this paper is as follows. In Section 2, we introduce the model, and briefly summarize its conformal properties based on the Bethe ansatz solution. In Section 3, we then obtain correlation exponents as functions of the mass and the momentum of the mobile impurity. The results obtained are then applied to the FES problem in quantum wires. We show that the critical exponent for the X-ray absorption depends on the mass of the core hole, in contrast to the previous findings of Ogawa *et al.* [5]. In Section 4, we outline a SU(ν) generalization of our analysis based on the g-on description of 1D correlated electrons. Brief summary is given in Section 5.

2 Critical properties of 1D electron systems with a mobile impurity

In this section, we study how a mobile impurity affects low-energy properties of 1D electron systems, by exploiting conformal field theory (CFT) analysis [28]. Let us start with a model for 1D correlated electron systems with a mobile impurity,

$$\mathcal{H} = -\frac{1}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} - \frac{1}{2m'}\frac{\partial^2}{\partial x'^2} + u\sum_{j>i=1}^{N}\delta(x_i - x_j) + v\sum_{i=1}^{N}\delta(x_i - x'), \qquad (1)$$

* Dedicated to J. Zittartz on the occasion of his 60th birthday

[a] e-mail: kawakami@primrose.ap.eng.osaka-u.ac.jp

in periodic boundary conditions, where m and x_i (m' and x') represent the mass and the coordinate of host electrons (a mobile impurity), respectively. This integrable model was originally introduced by Li and Ma [29] for a 1D boson system. We here solve this model for correlated electron systems with internal SU(2) spin symmetry in order to apply the results to the FES problem in quantum wires. It is assumed that the mobile impurity does not possess spin, so that it affects only the charge degrees of freedom.

Following Castro Neto and Fisher [23], we transform the Hamiltonian by the unitary transformation which converts the coordinates of electrons into those relative to a mobile impurity,

$$\mathcal{H} = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial y_i^2} - \frac{1}{2\mu}\left(\frac{\partial}{\partial y_0} - \sum_{i=1}^{N}\frac{\partial}{\partial y_i}\right)^2 + u\sum_{j>i=1}^{N}\delta(y_i - y_j) + v\sum_{i=1}^{N}\delta(y_i), \tag{2}$$

where μ is the reduced mass defined by $\mu = m'/m$.

Here we briefly summarize the techniques of Li and Ma [29], which diagonalize the above Hamiltonian *via* the Bethe ansatz method. We first note that because of translational invariance for y_0, the wave function of $N+1$ particles, $\psi(y_0, y_1, ..., y_N)$ can be written as $e^{iPy_0}\phi(y_1, y_2, ..., y_N)$, where P is a constant, *i.e.* the total momentum of the system. Therefore if we consider the transformed Hamiltonian (2) to act on this wave function, we can replace the impurity kinetic energy with

$$-\frac{1}{2\mu}\left(iP - \sum_{i=1}^{N}\frac{\partial}{\partial y_i}\right)^2. \tag{3}$$

Our problem is now effectively reduced to the N-particle problem for a given total momentum P. Following a standard Bethe-ansatz method, we now write down the N-particle wave function as a superposition of plane waves characterized by the charge rapidity k_j ($j = 1, 2, ..., N$). The scattering matrices were already obtained by Li and Ma [29]. For example, the scattering matrix between two host particles has an ordinary form

$$S_{ij} = \frac{k_i - k_j - iu}{k_i - k_j + iu} \tag{4}$$

for the singlet sector, whereas the scattering matrix between host particle and impurity is obtained as [29],

$$S_{i0} = \frac{k_i - \lambda/\mu - iv}{k_i - \lambda/\mu + iv}, \tag{5}$$

where we have introduced the impurity rapidity λ defined by $\lambda = P - \sum_j k_j$. These scattering matrices [29] have a desirable form for the Yang-Baxter relation [30] (see also discussions for a related impurity system [20]). By including the spin degrees of freedom for host electrons following Yang's method [30], we arrive at the algebraic Bethe equations for the charge rapidity k_j and the spin rapidity Λ_α,

$$k_j L = 2\pi I_j - \phi_i(k_j) + \sum_{\alpha=1}^{N_s}\theta(k_j - \Lambda_\alpha), \tag{6}$$

$$\sum_{j=1}^{N}\theta(\Lambda_\alpha - k_j) = 2\pi J_\alpha + \sum_{\beta=1}^{N_s}\theta((\Lambda_\alpha - \Lambda_\beta)/2) \tag{7}$$

with $\theta(k) = 2\tan^{-1}(2k/u)$, where N and N_s represents the number of total electrons and that of down spins, and L is the length of the system. Quantum numbers I_j and J_α satisfy the selection rules for electron systems, $I_j = (N_s+1)/2$ (mod 1), $J_\alpha = (N+N_s+1)/2$ (mod 1). The effect of the impurity manifests itself *via* the phase shift function $\phi_i(k) = 2\tan^{-1}(2(k-\lambda/\mu)/v)$. The total energy can be written as $E = \sum_j k_j^2/2 + \lambda^2/2\mu$. Note that the bare momentum (rapidity) of a mobile impurity λ should be determined consistently with the set of $\{k_j\}$ to preserve the total momentum conservation, $P = \sum k_j + \lambda$ (see discussions in Sect. 3 for the renormalized momentum which may be observable in experiments). Most formulae presented in this section are standard in the Bethe solvable models, so that we briefly summarize the main points necessary for the following discussions.

We first write down the finite-size corrections of the energy to examine conformal properties [31,32]. According to the Euler-Maclaurin expansion [33–35], we get the finite-size corrections to the ground state energy which are typical for $c = 1$ Gaussian CFT,

$$E \simeq L\varepsilon_\infty + E_s - \frac{\pi v_c}{6L} - \frac{\pi v_s}{6L}, \tag{8}$$

where v_c and v_s are the velocities of elementary excitations for charge and spin sectors, respectively. Note that the impurity effect causes the non-universal correction of order of unity,

$$E_s = \frac{\lambda^2}{2\mu} + \int_{-k_0}^{k_0}\frac{1}{2}k^2\rho_c^{(i)}(k)dk, \tag{9}$$

where we have introduced the density function for the charge rapidity $\rho_c(k)$ and the spin rapidity $\rho_s(\Lambda)$, and divided them into host and impurity parts; *e.g.* for the charge sector $\rho_c(k) = \rho_c^{(h)}(k) + \frac{1}{L}\rho_c^{(i)}(k)$. The impurity contribution to the density functions is determined by

$$\rho_c^{(i)}(k) = \frac{1}{2\pi}\rho_0(k - \lambda/\mu) + \int_{-\Lambda_0}^{\Lambda_0}\frac{d\Lambda'}{2\pi}K_1(k-\Lambda')\rho_s^{(i)}(\Lambda'), \tag{10}$$

$$\rho_s^{(i)}(\Lambda) = \int_{-k_0}^{k_0}\frac{dk'}{2\pi}K_1(\Lambda - k')\rho_c^{(i)}(k') - \int_{-\Lambda_0}^{\Lambda_0}\frac{d\Lambda'}{2\pi}K_2(\Lambda-\Lambda')\rho_s^{(i)}(\Lambda'), \tag{11}$$

where k_0 and Λ_0 denote the Fermi points of the charge and spin rapidities. For simplicity, we have introduced the functions,

$$\rho_0(k) = \frac{v}{(v/2)^2 + k^2}, \;\; K_n(k) = \frac{nu}{(nu/2)^2 + k^2}.$$

Note that the above E_s is nothing but the renormalized kinetic energy of a mobile impurity, which is referred to the surface energy in CFT. When we apply the present result to the X-ray absorption problem, the quantity E_s corresponds to the energy for a core-hole created by photon (see next section).

Similarly, the finite-size corrections to the excitation energy are obtained in a standard form [35],

$$\Delta E = \frac{2\pi v_c}{L}(\Delta_c^+ + \Delta_c^-) + \frac{2\pi v_s}{L}(\Delta_s^+ + \Delta_s^-), \qquad (12)$$

from which we can read the holomorphic and anti-holomorphic conformal dimensions for spin (s) and charge (c) sectors,

$$\Delta_c^{\pm} = \frac{1}{2}\left\{ \frac{\Delta N_c - n_{imp}^{(c)}}{2\xi} \pm \xi\left(\Delta D_c + \frac{\Delta D_s}{2} - d_{imp}^{(c)}\right)\right\}^2, \qquad (13)$$

$$\Delta_s^{\pm} = \frac{1}{4}\left(\Delta N_s - \frac{\Delta N_c}{2} \pm \Delta D_s\right)^2, \qquad (14)$$

for zero magnetic field. Here ΔN_c (ΔN_s) and ΔD_c (ΔD_s) label quantum numbers for charged and current excitations for the charge sector (spin sector) [35]. The quantities $n_{imp}^{(c)}$ and $d_{imp}^{(c)}$ are the impurity phase shifts obtained as

$$n_{imp}^{(c)} = \int_{-k_0}^{k_0} \rho_c^{(i)}(k)dk, \qquad (15)$$

$$d_{imp}^{(c)} = -\frac{1}{2}\int_{k_0}^{\infty} \rho_c^{(i)}(k)dk + \frac{1}{2}\int_{-\infty}^{-k_0} \rho_c^{(i)}(k)dk, \qquad (16)$$

where the Fermi point of spin rapidity Λ_0 is set to be infinity for zero magnetic field. Note that ξ is the dressed charge [34,35] determined by $\xi = \xi(k_0)$ with

$$\xi(k) = 1 + \frac{1}{2\pi}\int_{-k_0}^{k_0} \bar{K}(k-k')\xi_(k')dk', \qquad (17)$$

$$\bar{K}(k) = \int_{-\infty}^{\infty} \frac{e^{ikx}}{e^{u|x|}+1}dx. \qquad (18)$$

It is seen that the conformal dimension (13) for the charge sector takes a form typical for U(1) Gaussian CFT in which the parameter ξ features $c = 1$ critical line. On the other hand, the conformal dimension (14) for the spin sector is characteristic of SU(2) Kac-Moody theory for which the corresponding dressed charge takes a special value $1/\sqrt{2}$ (SU(2) enhancement point on $c = 1$ critical line). The calculational detail for these points can be found in the literatures [35,37]. The effect of a mobile impurity is incorporated only in the charge sector in (13) because we are now concerned with the impurity without spin degrees of freedom. The ordinary TL liquid parameter K_ρ which controls the correlation exponents [1,36] is given by $K_\rho = \xi^2/2$.

In the above expressions for conformal dimensions, most distinct from those for a static impurity is that it possesses the non-trivial phase shift $d_{imp}^{(c)}$ which does not appear in an ordinary static impurity, and is inherent in a mobile impurity. It is to be noticed, however, that the phase shift $d_{imp}^{(c)}$ can be observed even in the static impurity problem if the external magnetic flux is introduced in the ring geometry [20]: the excitation spectrum for such an impurity system with flux has the same structure as obtained in (12). However, a remarkable point is that the phase shift $d_{imp}^{(c)}$ in the present model is dynamically generated by the motion of the impurity. Namely, although in the original Hamiltonian (1) the impurity scattering v itself cannot produce the asymmetric phase shift, it naturally appears if the impurity motion is taken into account. This phase shift generated dynamically by the motion of the impurity should be distinguished from that caused by a static external flux mentioned above [20], although both models exhibit the similar excitation energy. In particular, both of the phase shifts $n_{imp}^{(c)}$ and $d_{imp}^{(c)}$ in our system depend on the microscopic parameters of the system such as the impurity scattering v, the electron-electron interaction u, the electron density n, *etc.* Furthermore, when we apply our impurity model to the X-ray absorption problem, for which the mobile impurity is assumed to be created suddenly, the above non-trivial phase shifts can indeed show up in the physical quantities. As will be discussed in the next section in detail, when the X-ray absorption creates the impurity (core-hole) with a given momentum in our system, this produces the asymmetric phase shift, which gives rise to the anomalous exponent for the absorption spectrum. In contrast, in the case of the ordinary impurity model with an external flux, the asymmetric phase shift appears in the excitation spectrum, but should not show up in the conformal dimensions or the correlation exponents, as pointed out by Affleck and Ludwig [7]. In this way, in order to discuss the anomalous critical exponents, it is quite important to notice that our asymmetric phase shift $d_{imp}^{(c)}$ is dynamically generated by the motion of the impurity. It will be indeed shown momentarily that the two phase shifts $n_{imp}^{(c)}$ and $d_{imp}^{(c)}$ in our case play a crucial role to determine the long-time behavior of correlation functions.

This completes our preparation for computing the correlation exponents related to the FES problem. Before concluding this section we wish to mention one thing about a mobile impurity. Although the impurity in our system is mobile, there may naturally arise a question whether a mobile impurity is dynamically localized or not. This problem was previously addressed by Prokof'ev [21], and Castro Neto and Fisher [23]. According to their analyses, for the system with repulsive δ-function interaction among electrons, the impurity cannot be localized, which is consistent with the present results.

3 Application to Fermi-edge singularity problem

We now wish to determine the critical exponents for correlation functions related to the FES problem. To this end, let us recall here that conformal dimensions (13) have the same form as expected for the system in which two different twisted boundary conditions are imposed on the left- and right-going electrons with twist angles $\delta_L = \frac{1}{2}(n_{imp}^{(c)} - 2d_{imp}^{(c)})$ and $\delta_R = \frac{1}{2}(n_{imp}^{(c)} + 2d_{imp}^{(c)})$ in unit of π. It is known that these phase shifts can be effectively eliminated [7], if we consider the static correlation functions. The non-trivial exponents we are now interested in should appear in the FES problem for which an impurity is suddenly created at time $t = 0$. The long-time behavior of correlation functions is then controlled by the screening effects due to conduction electrons, resulting in non-trivial FES exponents.

We first observe the long-time behavior of the local one-electron Green function, $G(t) \leq \psi_\sigma^\dagger(t)\psi_\sigma(0) \gtrsim t^{-\alpha_G}$ with $\sigma = \uparrow, \downarrow$. To obtain the corresponding critical exponent α_G, we should take quantum numbers which increase the number of host electrons by one [37]. In this condition, the most relevant set of quantum numbers turns out to be given by $(\Delta N_c, \Delta N_s, \Delta D_c, \Delta D_s) = (1, 1, 0, \pm 1/2)$. Moreover, one can see that the condition $d_{imp}^{(c)} < 0$ always holds for $\lambda/\mu > 0$. Therefore, by choosing $(\Delta N_c, \Delta N_s, \Delta D_c, \Delta D_s) = (1, 1, 0, -1/2)$ for down-spin electron, the critical exponent is evaluated as,

$$\alpha_G = \frac{(1 - n_{imp}^{(c)})^2}{2\xi^2} + 2\xi^2 \left(\frac{1}{4} + d_{imp}^{(c)}\right)^2 + \frac{1}{2}. \quad (19)$$

Note that the critical exponent for up-spin electron is determined by $(\Delta N_c, \Delta N_s, \Delta D_c, \Delta D_s) = (1, 0, \pm 1/2, \mp 1/2)$, which gives the same result as (19). Also, the critical exponent for the overlap-integral (orthogonality-catastrophe exponent) [38] is obtained by setting all the quantum numbers zero,

$$\alpha_O = \frac{(n_{imp}^{(c)})^2}{2\xi^2} + 2\xi^2 \left(d_{imp}^{(c)}\right)^2. \quad (20)$$

This correlation exponent may be relevant to the spectral function for core-electron photoemission in quantum wires, for which the number of conduction electrons is not changed in photoemission process. We plot two correlation exponents as a function of λ/μ in Figure 1, and the phase shifts $n_{imp}^{(c)}$ and $d_{imp}^{(c)}$ in Figure 2. We find that the correlation exponents show a non-monotonic behavior when effective impurity position λ/μ is close to the Fermi level, because the impurity phase shifts change their values rather sharply around the Fermi level as seen in Figure 2. Here, some comments are in order for the impurity rapidity λ. As mentioned in the introduction, the momentum λ is the rapidity for the impurity, which should be determined consistently with the condition $P = \lambda + \sum_j k_j$. Therefore, when the impurity is created suddenly, it also induces the change in $\sum_j k_j$ for conduction electrons *via* the interaction, which is evaluated as $2k_F d_{imp}^{(c)}$. Then the total momentum which may be observable in experiments is given by $P = \lambda - 2k_F d_{imp}^{(c)}$; P is also regarded as the renormalized momentum of the impurity if the impurity is created when host electrons are in the ground state. To make the relation between λ and P explicit, we have plotted the total momentum P as a function of λ in Figure 3.

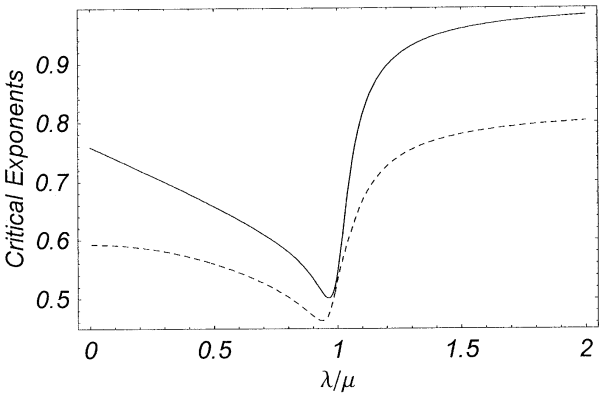

Fig. 1. Critical exponents α_G and α_O as a function of λ/μ: the solid line (dashed line) represents α_G (α_O). The parameters for interactions u and v are chosen as $v/u = 0.1$. The cut-off parameter for rapidities is set to be $k_0 = 1.0$.

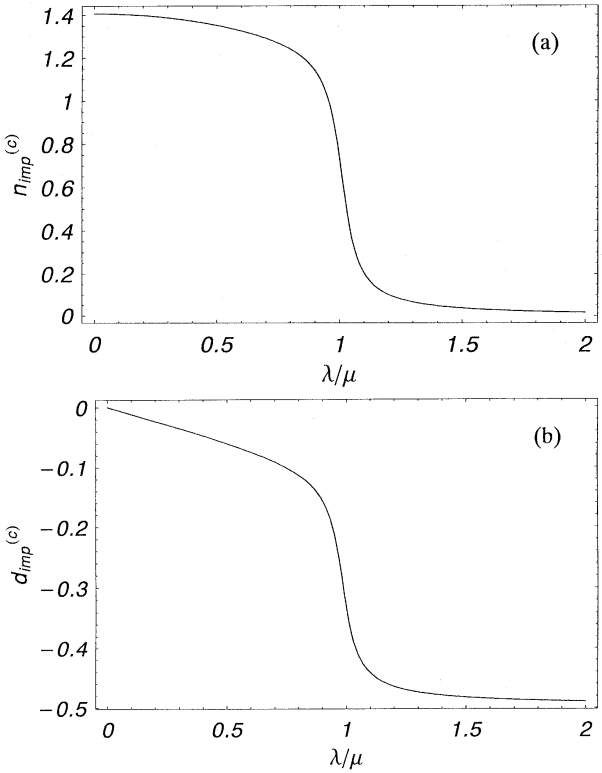

Fig. 2. Phase shifts (a) $n_{imp}^{(c)}$ and (b) $d_{imp}^{(c)}$ as a function of λ/μ. The parameters used are the same as those in Figure 1.

Here, it may be also instructive to give some discussions for the orthogonality catastrophe (OC) exponent α_O introduced above. Rosch and Kopp [22], and Castella

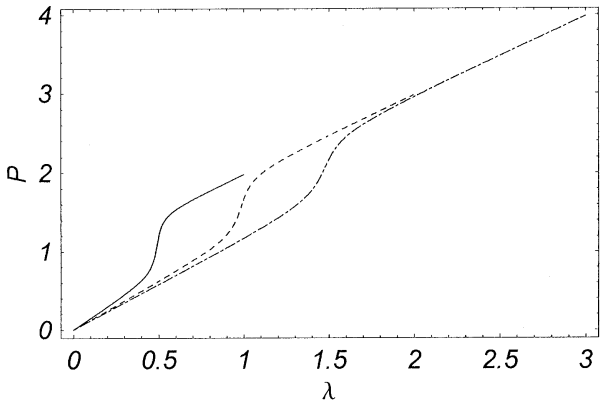

Fig. 3. Plots of the total momentum P as a function of the impurity rapidity λ for several choices of the impurity mass μ: the solid, the dashed and the dash-dotted lines correspond to the $\mu = 0.5$, 1 and 1.5 cases. The parameters used are the same as those in Figure 1.

[26] already studied the propagator for a mobile impurity in the noninteracting fermionic environment, from which they obtained the corresponding OC exponent. For example, Rosch and Kopp obtained $(2/3)(n_{imp}^{(c)})^2$ in the heavy mass limit. In contrast to their treatment, the impurity state in the present case is assumed to be created at an excited state with the total momentum P and the renormalized energy $E_s(P)$ in equation (9), which is an eigenstate of the system. Therefore our OC exponent obtained from CFT analysis of this excitation spectrum is different from theirs, and may be related to the overlap integral between the ground state (without impurity) and such an excited eigenstate (with impurity). In fact, when we take the limit of the heavy mass, α_O leads to $(n_{imp}^{(c)})^2$ for the noninteracting case, which is different from the OC exponent of Rosch and Kopp, $(2/3)(n_{imp}^{(c)})^2$

Furthermore, since we are considering an excited eigenstate with the total momentum P, the propagator for the renormalized impurity shows a power-law behavior with the exponent α_O even for finite P, although this is not the case for the impurity propagator discussed by Rosch and Kopp [22]. It is still open to relate these two exponents explicitly, which should be resolved in the future study.

We are now ready to apply our analysis to the FES problem in X-ray absorption in quantum wires. As mentioned before, recent optical experiments for quantum wires pointed out the importance of mobile core hole created by photon [24]. In fact, this problem has been theoretically studied by several authors [5,25,26]. In order to discuss the FES problem in quantum wires, we first regard a mobile impurity as a mobile core hole which is created by X-ray absorption. Therefore, the renormalized momentum of the mobile impurity, $P = \lambda - 2k_F d_{imp}^{(c)}$, is now regarded as that of the mobile core hole, which should be set to be k_F. The critical exponent $\beta_G = \alpha_G - 1$ for the FES (with frequency dependence $\sim \omega^{\beta_G}$) is thus given directly by that for the one-particle Green function shown in Figure 2. It is seen that the exponent β_G has a negative value irrespective of the momentum, which means that we have the divergence FES in the range of λ/μ shown in Figure 1. As a natural consequence, the critical exponent for the FES should depend on the mass of the core hole created in quantum wires. Note that such dependence shows up *via* the non-trivial phase shifts $n_{imp}^{(c)}$ and $d_{imp}^{(c)}$.

We now compare our result with that obtained by Ogawa *et al.* [5]. By exploiting bosonization method with some other approximations, they claimed that the FES exponent may not depend on the mass of the core hole created. Apparently, our result disagrees with theirs. This discrepancy was also pointed out by Castella *et al.* by using the Hubbard model with only one down-spin electron [25,26]. Here we give a possible account for this discrepancy. We first note that our formulae for conformal dimensions (13) include two phase shifts, $n_{imp}^{(c)}$ as well as $d_{imp}^{(c)}$. The former effect has been already taken into account previously [5], but the latter is inherent in our mobile impurity. The appearance of $d_{imp}^{(c)}$ implies that there should effectively exist an asymmetric forward scattering like

$$\mathcal{H}_{imp} = V(\lambda)\delta(x)[\rho_L(x) - \rho_R(x)], \tag{21}$$

for a given λ, where $\rho_L(x)$ ($\rho_R(x)$) is the density of left-going (right-going) electrons. Here we have used the relative-coordinate representation introduced in (2). Note that $\mathcal{H}_{imp}$ is the impurity potential coupled to the electron *current* (not the total density). This type of asymmetric scattering is characteristic of the mobile impurity, whose origin is not clearly seen in the original Hamiltonian (1), but is naturally understood in the transformed Hamiltonian (2). Namely, one can see the gauge potential explicitly included in the kinetic energy of the impurity, which may give rise to the coupling of the impurity to electron currents. In the approach of Ogawa *et al.* [5], this asymmetric scattering term has not been included, which may be a reason why they could not find the mass dependence. So, in order to treat a mobile impurity, it is important to take into account the asymmetric forward scattering due to impurity.

4 Multicomponent systems with a mobile impurity

We now wish to extend our analysis to general multicomponent cases, including non-integrable models. To this end, it is useful to exploit the notion of g-ons, which can describe the universal low-energy properties of 1D correlated electron systems phenomenologically in a simplified manner. Let us start with a brief introduction of g-ons defined in exclusion statistics [39,40]. Exclusion statistics is based on counting the change of the dimension of the one-particle Hilbert space when a particle is added to the system, which is explicitly formulated as [39],

$$\frac{\partial D_\alpha(k_\alpha)}{\partial N_\beta(k'_\beta)} = -g_{\alpha\beta}(k_\alpha - k'_\beta), \tag{22}$$

where $D_\alpha(k_\alpha)$ and $N_\alpha(k_\alpha)$ are the numbers of unoccupied (hole) and occupied (particle) states specified by the internal quantum numbers $\alpha = (1, 2, ..., \nu)$ and corresponding momentum k_α. The matrix $g_{\alpha\beta}$, which is called statistical interaction, describes correlation effects among particles. Simple cases $g_{\alpha\beta}(k_\alpha - k'_\beta) = g\delta_{\alpha\beta}\delta_{k_\alpha k'_\beta}$ with $g = 1$ and $g = 0$ correspond to free fermions and free bosons, respectively, and for general fractional value g, we call g-ons which obey ideal exclusion statistics [39,40]. An important point is that g-ons are idealized particles which can describe essential properties of 1D correlated electron systems in the low-energy regime: the statistical parameter g serves as the TL liquid parameter. So far, the impurity effect on g-ons has not been discussed. We wish to study g-ons with a mobile impurity in order to generalize our discussions in the previous section.

Let us begin with a simple example. Consider 1D noninteracting Dirac fermions with a mobile impurity. The spectrum of the system may be given by

$$k_j L = \phi_i(k_j) + 2\pi I_j, \tag{23}$$

where $\phi_i(k_j)$ is the phase shift due to a mobile impurity (see Eq. (7)). Now let us turn on the statistical interaction p. It is known that the statistical interaction between particles can be introduced by turning on the step-wise two-body phase shift [39,40] $p\pi\mathrm{sgn}(k_j - k_l)$, so that

$$k_j L = \phi_i(k_j) + 2\pi I_j + p\pi \sum_l \mathrm{sgn}(k_j - k_l). \tag{24}$$

In this case the total statistical interaction reads $g = 1 + p$, which is related to the TL liquid parameter as $K_\rho = 1/(2g - 1)$. Notice that if $p = 0$ the system reduces to free fermions with statistical interaction $g = 1$. Thus, the above Bethe-type equations can describe one-component TL liquids with a mobile impurity in terms of g-ons.

Following Sutherland's method [41], we now generalize the above formulation to 1D multicomponent electron systems with SU(ν) spin symmetry by introducing ν-kind of rapidities $k_j^{(\alpha)}$. We thus arrive at the Bethe-type equations for SU(ν) systems with a mobile impurity,

$$k_j^{(1)} L = \phi_i(k_j^{(1)}) + 2\pi I_j^{(1)} + \sum_{m=1}^{M_2} \Phi(k_m^{(2)} - k_j^{(1)}) + p\sum_{l=1}^{M_1} \Phi(k_j^{(1)} - k_l^{(1)}), \tag{25}$$

$$\sum_{l=1}^{M_\alpha} \Phi(k_m^{(\alpha)} - k_l^{(\alpha)}) + 2\pi I_m^{(\alpha)} = \sum_{s=\pm 1} \sum_{j=1}^{M_{\alpha+s}} \Phi(k_m^{(\alpha)} - k_j^{(\alpha+s)}), \tag{26}$$

for $2 \le \alpha \le \nu$, where $\Phi(k) = \pi\mathrm{sgn}(k)$ and $I_j^{(\alpha)}$ is an integer or a half integer which classifies the charge and spin excitations. Note that the statistical interaction p is introduced only for the charge sector to preserve SU(ν) spin symmetry. In the above equations we have defined the quantity $M_\alpha = \sum_{\beta=\alpha}^{\nu} N_\beta$ where N_β is the number of electrons with spin β. The energy is assumed to have the form $E = \sum_j (k_j^{(1)})^2/2 + \lambda^2/2\mu$.

Let us write down the excitation spectrum. Using the above equations, low-energy excitations are classified as $\Delta E = 2\pi v x_b/L$, with the scaling dimension x_b obtained in the matrix formula,

$$x_b = \frac{1}{4}\mathbf{m}^t\mathbf{T}\mathbf{m} + \mathbf{d}^t\mathbf{T}^{-1}\mathbf{d}, \tag{27}$$

for zero magnetic field, where the $\nu \times \nu$ matrix $\mathbf{T}$ is evaluated as,

$$\mathbf{T} = \begin{pmatrix} g & -1 & & \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{pmatrix} \tag{28}$$

with $g = 1 + p$. Here the ν-component vector $\mathbf{m}$ consists of quantum numbers which classify the charge and spin excitations, where we have assumed that m_1 and d_1 label charge excitations [42,43]. Note that these quantum numbers include the effect of the phase shifts, *e.g.* $m_\alpha = \Delta M_\alpha + [1 + (1 - \alpha)/\nu]n_{imp}^{(c)}$, where ΔM_α is an integer whereas $n_{imp}^{(c)}$ denotes the impurity phase shift. Similar expression holds for d_α: $d_\alpha = \Delta D_\alpha + d_{imp}^{(c)}\delta_{1\alpha}$. The selection rule for the quantum numbers read [42,43]: $\Delta D_\alpha = \frac{1}{2}(\Delta M_{\alpha-1} + \Delta M_{\alpha+1})$ mod 1, with $\Delta M_0 = \Delta M_1$ and $\Delta M_{\nu+1} = 0$. Note that the critical behavior of the present multicomponent model is described by shifted U(1) CFT for charge sector and level-1 SU(ν) Kac-Moody theory for spin sector.

Now, we can read the critical exponents for the FES problem. We recall here again that in order to obtain nontrivial exponents, we should consider the situation where an impurity is suddenly created at $t = 0$. By generalizing the arguments in the previous section, we first consider the long-time behavior of the electron Green function. This is controlled by the quantum numbers which add one electron to the system. Thus by setting $\Delta M_\alpha = 1$, and $\Delta D_{\alpha\pm 1} = \mp 1/2$, the critical exponent for the FES is computed as,

$$\alpha_G = \frac{1 + \nu p}{2\nu}\left(1 - n_{imp}^{(c)}\right)^2 + \frac{2}{\nu(1 + \nu p)} \times \left(\frac{1}{2} + \nu d_{imp}^{(c)}\right)^2 + \frac{\nu - 1}{\nu}. \tag{29}$$

The TL liquid parameter is now identified with $K_\rho = 1/(1 + \nu p)$, if it is normalized as $K_\rho = 1$ for noninteracting case, as usual [1,36]. Note that the value $(\nu - 1)/\nu$ in the above formula is nothing but the spin exponent determined from the level-1 SU(ν) Kac-Moody theory. Also, the critical exponent for the orthogonality catastrophe reads,

$$\alpha_O = \frac{1 + \nu p}{2\nu}\left(n_{imp}^{(c)}\right)^2 + \frac{2\nu}{(1 + \nu p)}\left(d_{imp}^{(c)}\right)^2, \tag{30}$$

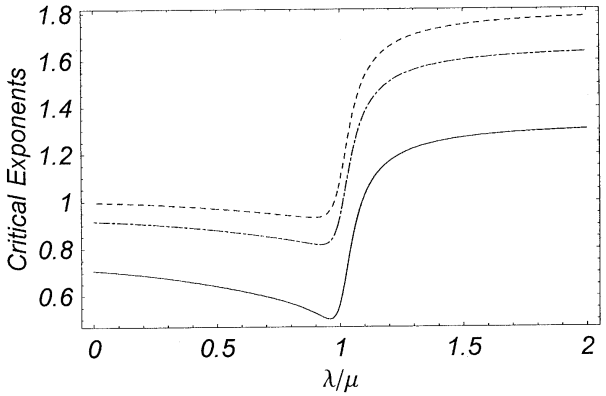

Fig. 4. Critical exponent α_G for SU(ν) case as a function of λ/μ with the statistical interaction $g = 2$: the solid, the dash-dotted and the dashed lines correspond to the ν=2, 5 and 10 cases. The phase shifts are taken from those shown in Figure 2.

for which the spin exponent does not show up because the spin excitation is not relevant to this correlation function. For the $\nu = 2$ case (SU(2) electrons), we can see that the above formulae exactly correspond to those obtained in the previous section with the relation $\xi^2 = 2/(1+2p)$. We show the results for the FES exponent α_G for the SU(ν) case in Figure 4. It is seen that the mass dependence of the correlation exponents for the SU(ν) case exhibits a behavior similar to the SU(2) case *via* the variation of two kind of phase shifts. However, the value of the exponent itself becomes large as ν is increased, and the FES exponent $\beta_G = \alpha_G - 1$ relevant to the frequency dependence can take a positive value in a certain momentum region, resulting in the convergence FES. We can thus say that the large spin degeneracy has a tendency to suppress the FES in general. We wish to note that the merit of our analysis based on g-ons is that the formulae obtained are phenomenologically extended to more general cases including non-integrable models. For example, if the diagonal elements of the matrix (28) are continuously changed, we can treat the case with spin anisotropy, for which the exact solution may not be easily obtained.

5 Summary

In this paper, the critical properties for the 1D correlated electron system which includes a mobile impurity have been studied by means of the Bethe ansatz method and conformal field theory. We have then applied the results to the FES problem in quantum wires, and have shown that the FES exponent indeed depends on the mass of the core hole created by the X-ray absorption. The generalization to the SU(ν) electron systems have been discussed by using the g-on description of 1D electron systems. Our prediction for the FES exponent is based on CFT analysis of the excitation spectrum. So, it is desirable to calculate the correlation function directly to confirm our conclusion, which is now under consideration.

To conclude the paper, we wish to mention that the problem in a mobile impurity in 1D electron systems is also related to that for the photoemission in the Mott insulator, as firstly pointed out by Sorella and Parola [44,45]. To describe various spectral properties for Mott insulators as well as for quantum wires in a unified fashion is an interesting future problem.

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